CORRIGENDUM TO "PHASE TRANSITIONS IN A PIECEWISE EXPANDING COUPLED MAP LATTICE WITH LINEAR NEAREST NEIGHBOUR COUPLING"

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The proofs in our paper [1] need two corrections that we present below. The results remain valid as published. The first correction concerns the order in which some parameters in our construction have to be chosen; the second one fills a gap in the argument which extends the results from piecewise linear Markov maps to smooth circle maps.

1. Choice of the parameter γ

The constant β appearing in **lemma 2.2.(a)**[1] has to be taken independently of γ, contrarily to what is written in the published version. The same has to be checked for the corresponding constant B in **lemma 4.1** and in **(5.3)**.

We need the independence of β and B from γ to perform the computation at the end of **section 5 (pp 2205–2206)**: since the constants K_1 and K_2 depend only on B, $K = 3(K_1+\frac{1}{2}(1+K_2)\beta)$ can in this case be chosen independently of γ and k. This is essential to conclude because the values of κ such that $\kappa K \frac{1}{2} \operatorname{Var}(h_0) \leq \Delta^8$ depend (trivially) on K. Once the value of κ fixed, one has to choose γ small satisfying **lemma 2.2.(c)**, then k large satisfying **lemma 2.2.(a-b)**, without modifying K.

One can indeed fix β independently of γ thanks to the facts that $\tilde\tau$ is a piecewise affine Markov mapand that γ does not appear in the lengths of images of its affinity intervals. Indeed, choose for any $\kappa > 0$ a k such that $\inf |(\tilde\tau^k)'| \geq \frac{2v}{\kappa}$. Since $\tilde\tau$ is piecewise affine the classical proof of Lasota-Yorke inequality (see for example proposition 2.1 in [2]) may be simplified to give

$$\operatorname{Var}(P_{\tilde\tau}(f)) \leq v \operatorname{Var}(P_{\tilde\tau^k}(f)) \leq \kappa \operatorname{Var}(f) + \frac{2v}{\min_i |\tilde\tau^k(I_i)|} \int |f|\, dm\,, \tag{1.1}$$

where $\{I_i\}_i$ are the affinity intervals of $\tilde\tau^k$. But, as the affinity intervals of $\tilde\tau$ form a Markov partition for $\tilde\tau$, all intervals $\tilde\tau^k(I_i)$ occur as images $\tilde\tau(J)$ of affinity intervals J of $\tilde\tau$. Then an inspection of the specific form of the map $\tilde\tau$ shows that the constant

$$\beta = \frac{2v}{\min_i |\tilde\tau^k(I_i)|} = \frac{2(1-\eta)}{\delta} \tag{1.2}$$

depends on η and δ but is independent from γ and k. We can then conclude for the tensor product as in lemma 3.2 of [2], and for B as previously, since the modification of β that results in B will only be due to the coupling.

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[1] All the boldfaced references are related to the published version of the paper.

2. The case of smooth modifications

The formulation of **remark 2.3** is a bit misleading and the proof of **theorem 1 for the map $\bar{\tau}^3$** in **section 7** incomplete. We use in fact the specific construction of the local map in the proof of the exponential estimate from **section 5**, and it can not be directly transfered to a smooth modification of it.

More specifically, the inclusion **(5.12)** is obtained thanks to the calculations of **section 3**, in particular the two summarized observations at its end. These observations remain valid for $\check{\tau} = \frac{1}{v}\acute{\tau}^k$, since $\acute{\tau}$ has the same Markov partition structure as $\tilde{\tau}$, but cannot be checked for a perturbation $\bar{\tau}$ such that

$$\rho := |||P_{\bar{\tau}} - P_{\check{\tau}}||| = \sup\left\{\int |(P_{\bar{\tau}} - P_{\check{\tau}})(f)|\,dm : \ \mathrm{Var}(f) \leq 1\right\} \tag{2.1}$$

is small.

This difficulty can however be overcome, since formula **(5.12)** is only used in the "probabilistic" computation **(5.13)**. We explain below how this computation has to be modified when the local map $\bar{\tau}$ is a smooth modification of $\check{\tau}$. For all notations which are taken from the published version, we assume that the local map is $\bar{\tau}$. We denote also $\bar{T}$ (resp. $\check{T}$) the 4-fold direct product of $\bar{\tau}$ (resp. $\check{\tau}$).

We want to compute

$$\int_{B_0(\boldsymbol{x}_{\Pi(\boldsymbol{i})})} h_0(y)\,dy = \int_{B_1(\boldsymbol{x}_{\Pi(\boldsymbol{i})})} P^3_{\bar{T}} P_{\Phi_\epsilon^{\{t_1\}}}\Big(1_{\bar{\Gamma}_{(\boldsymbol{i},t_1)}}\, P^3_{\bar{T}} P_{\Phi_\epsilon^{\{t_1-1\}}} P^{t_1-1}_{\epsilon|\boldsymbol{x}_{\Pi(\boldsymbol{i})}} h_0\Big)(y)dy\,, \tag{2.2}$$

where

$$\bar{\Gamma}_{(\boldsymbol{i},t_1)} := \{\xi \in I^{U(\boldsymbol{i})} : (\boldsymbol{i},0) \text{ is an error site for } \bar{T}^3 \circ \Phi_\epsilon^{\{t_1\}} \text{ at } (\xi, T^{t_1}_{\epsilon,\Pi(\boldsymbol{i})}(\boldsymbol{x}_{\Pi(\boldsymbol{i})}))\}\,, \tag{2.3}$$

so that $E_{(\boldsymbol{i},t_1)} = T^{-t_1}_{\epsilon,\boldsymbol{x}_{\Pi(\boldsymbol{i})}}(\Gamma_{(\boldsymbol{i},t_1)})$. We then need to replace:

- the six terms $P_{\bar{T}}$ by $P_{\check{T}}$, making an error of the order of $\rho K_3 \,\mathrm{Var}(h_0)$, where K_3 is the sum of six uniform controls on norms of operators acting on BV spaces (since the operators $P_{\bar{T}}$ and $P_{\check{T}}$ satisfy a uniform Lasota-Yorke inequality).
- the term $1_{\bar{\Gamma}_{(\boldsymbol{i},t_1)}}$ by its equivalent for $\check{T}$, $\check{\Gamma}_{(\boldsymbol{i},t_1)} := \{\xi \in I^{U(\boldsymbol{i})} : (\boldsymbol{i},0)$ is an error site for $\check{T}^3 \circ \Phi_\epsilon^{\{t_1\}}$ at $(\xi, T^t_{\epsilon,\Pi(\boldsymbol{i})}(\boldsymbol{x}_{\Pi(\boldsymbol{i})}))\}$.
  We notice that $\bar{\Gamma}_{(\boldsymbol{i},t_1)}$ can be written as the disjoint union of 8 terms of the type $\bar{\Gamma}^{(1)}_{(\boldsymbol{i},t_1)} = \{\xi : \xi_{\boldsymbol{i}} \geq 0, \xi_{\boldsymbol{i}+e_1} \geq 0, \xi_{\boldsymbol{i}+e_2} \geq 0$ and $(\bar{T}^3 \circ \Phi_\epsilon^{\{t_1\}}(\xi))_{\boldsymbol{i}} \leq 0\}$. One can then evaluate the difference (denoting $\check{\Gamma}^{(1)}_{(\boldsymbol{i},t_1)}$ the equivalent term for $\check{T}$)

$$\begin{aligned}
&\int \Big|1_{\bar{\Gamma}^{(1)}_{(\boldsymbol{i},t_1)}} - 1_{\check{\Gamma}^{(1)}_{(\boldsymbol{i},t_1)}}\Big| h(\xi)\,d\xi \\
&\leq \int 1_{\{\xi_{\boldsymbol{i}} \leq 0\}} \Big|\big(P^3_{\bar{T}} - P^3_{\check{T}}\big) P_{\Phi_\epsilon^{\{t_1\}}}\big(1_{\{\xi_{\boldsymbol{i}} \geq 0, \xi_{\boldsymbol{i}+e_1} \geq 0, \xi_{\boldsymbol{i}+e_2} \geq 0\}} h\big)\Big|(\xi)\,d\xi \\
&\leq K_4 \rho \,\mathrm{Var}\, h
\end{aligned} \tag{2.4}$$

  thanks to (2.1). The error made when replacing $1_{\bar{\Gamma}_{(\boldsymbol{i},t_1)}}$ by $1_{\check{\Gamma}_{(\boldsymbol{i},t_1)}}$ in (2.2) is then of the order of $\rho\, 8 K_2 K_4 \,\mathrm{Var}(h_0)$.

We can now use the equivalent formulation of **(5.12)** for $\check{T}$,

$$\check{\Gamma}_{(\boldsymbol{i},t_1)} \subseteq \{\xi : \xi \notin G\} \cup \{\xi : \check{T}\Phi_\epsilon^{\{t_1\}}(\xi) \notin G\} \cup \{\xi : \check{T}^2\Phi_\epsilon^{\{t_1\}}(\xi) \notin G\}\ , \tag{2.5}$$

to compute the remaining integral

$$\int_{B_1(\boldsymbol{x}_{\Pi(i)})} P_{\check{T}}^3 P_{\Phi_\epsilon^{\{t_1\}}} \Big(1_{\check{\Gamma}_{(i,t_1)}}\, P_{\check{T}}^3 P_{\Phi_\epsilon^{\{t_1-1\}}} P_{\epsilon|\boldsymbol{x}_{\Pi(i)}}^{t_1-1} h_0\Big)(y)dy \tag{2.6}$$

exactly as in **(5.13)**. One can finally conclude by choosing κ and ρ such that

$$(\kappa K + \rho(K_3 + 8K_2K_4))\frac{1}{2}\,\mathrm{Var}(h_0) \leq \Delta^8\ . \tag{2.7}$$

References

[1] J.-B. Bardet, G. Keller, *Phase transitions in a piecewise expanding coupled map lattice with linear nearest neighbour coupling*, Nonlinearity, **19** (2006), 2193–2210.

[2] G. Keller, C. Liverani, *A spectral gap for a one-dimensional lattice of coupled piecewise expanding interval maps*, in: Dynamics of Coupled Map Lattices and of Related Spatially Extended Systems (Eds.: J.-R. Chazottes, B. Fernandez), Lecture Notes in Physics **671** (2005), pp. 115–151, Springer Verlag.

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